2. I. E. Rossiter, "Wind tunnel experiments on the flow over rectangular cavities at subsonic and transonic speeds," Am. Rocket Soc. RM 3438 (1966).
3. H. Heller and D. Bliss, "The physical mechanism of flow-induced pressure fluctuations in cavities and concepts for their suppression," AIAA Paper 75-491 (1975).
4. M. G. Morozov, "Self-excitation of oscillations in supersonic separation flows," Inzh.-Fiz. Zh., 27, No. 5 (1974).
5. A. J. Bilanin and E. E. Covert, "Estimation of possible excitation frequencies for shallow rectangular cavities," AIAA J., 11, No. 3, 347 (1973).
6. A. N. Antonov, V. K. Gretsov, and S. P. Shalaev, "Nonsteady supersonic flow over bodies with a front-mounted needle," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 5 (1976).
7. A. G. Munin and V. E. Kvitka (editors), Aviation Acoustics [in Russian], Mashinostroenie, Moscow (1973).
8. P. O. Davies, M. J. Fisher, and M. J. Barratt, "The characteristics of the turbulence in the mixing region of a round jet," J. Fluid Mech., 15 (1963).

ELECTROMAGNETIC METHOD OF MEASURING MASS VELOCITY AND ELECTRICAL CONDUCTIVITY WHICH VARY ALONG THE STREAM

A. V. Pinaev

UDC 53.082.7+538.4

1. A theory of velocity measurement in an MHD channel with allowance for nonuniformities of the magnetic field, mass velocity, and electrical conductivity across the channel is presented in [1, 2] in connection with the problem of the electromagnetic measurement of flow rate. The solution of the problem of the electric field distribution for a stream with a constant electrical conductivity and a mass velocity which varies along the channel is presented in [2]. A method of electrical contact measurements is presented in [3, 4] which gives a good enough resolution to obtain the profile of electrical conductivity which varies along the stream on the example of a detonation wave in a solid explosive, and an estimate of the accuracy of the method is given. A survey of electrical measurements of electrical conductivity is given in [5].

Noncontact methods of measurement, which are modifications of Lin's method [6], are unsuitable for measuring the electrical conductivity of a medium when the velocity varies along its stream. The electrical contact method [4] allows one to obtain the profile of electrical conductivity in a detonation wave with good spatial resolution, but it does not yield any data on the mass velocity of the stream.

In a number of practical problems, such as in the case of the investigation of shock and detonation waves, the dependence of the mass velocity and the electrical conductivity on the coordinate along the stream proves important. The accuracy of the MHD contact measurement of a mass—velocity profile was estimated in [7] and it was shown that MHD contact measurements of the profile of electrical conductivity are possible when the mass-velocity profile is not known in advance.

However, the MHD contact method is unsuitable for determining the mass-velocity profile when it varies significantly along the stream, such as in a detonation wave, since the error of the velocity measurement is large [7]. And for the same reason it is undesirable to use the method of electromagnetic measurements suggested in [8] to determine the profiles of mass velocity and resistance in detonation waves.

In the present report a method of contact electromagnetic measurements is described and the conditions for such measurements are found which allow one to eliminate the influence of the nonuniformity of the mass velocity and electrical conductivity on the accuracy of their determination.

2. A schematic diagram of the measurements is presented in Fig. 1. A medium with a mass velocity $v(z)$ and an electrical conductivity $\sigma(z)$, where z is the coordinate along the stream, propagates along the z_0 axis of a channel of circular cross section with conducting walls. A coaxial conductor 1 is fastened at the center of the channel. The channel consists of a cylindrical capacitor, the central 1 and outer plates 2 and 3 of which serve as the electrodes. The outer plate is formed by two metallic cylinders separated by an insulating spacer 4. The cylinders are electrically connected with each other by a connector 5.

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 2, pp. 98–103, March–April, 1981. Original article submitted January 28, 1980.

0021-8944/81/2202-0222$07.50

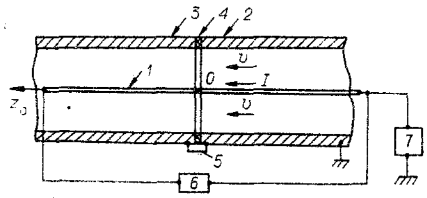

Fig. 1

A periodic (or constant) radial electric field and a constant (or periodic) magnetic field, the lines of which consist of cylindrical circles with centers along the axis of the tube, are produced in the channel by the outside electrical sources 6 and 7. With motion of the conducting medium from right to left in the cylinder 2 a current flows between the electrodes 1 and 2. At the moment corresponding to the arrival of the leading boundary of the conducting zone in the cylinder 3 a current starts to flow between the electrodes 1 and 3 through the electrically conducting medium and between the cylinders 2 and 3 through the connector. The size of this current grows up to the moment corresponding to the arrival of the trailing boundary of the conducting zone at the cylinder 3.

Let us set up the problem of determining the profiles of mass velocity v(z) and electrical conductivity $\sigma(z)$ using this scheme.

We will assume that the following conditions are satisfied:

1) τ, the characteristic time of variation of v(z) and $\sigma(z)$, considerably exceeds the half-period T/2 of variation of the external field, i.e., $\tau \gg T/2$;

2) the displacement current in the conducting stream can be neglected, i.e., $\varepsilon\omega\sigma^{-1} \ll 1$, where $\varepsilon \simeq \varepsilon_0 = 10^{-9}/36\pi$ F/m; ω is the cyclic frequency of the external field, radians per second;

3) self-induction in the stream can be neglected, i.e., $(\mu\sigma\omega)^{-1/2} \gg R$, where $\mu \simeq \mu_0 = 4\pi \cdot 10^{-7}$ G/m; R is the inner radius of the cylinder, meters;

4) Ohm's law for the medium is written in the form $\mathbf{j} = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B})$, i.e., the conductivity of the electrically neutral medium is isotropic and does not depend on the magnetic field $((eB/m)\tau_{co} \ll 1)$, where $\mathbf{j}$ is the current density in the medium, amperes per square meter, **E** and **B** are the electric field strength, volts per meter, and the magnetic field induction, teslas, in the stationary coordinate system; e and m are the electron charge, coulombs, and mass, kilograms; τ_{co} is the time between collisions of an electron with particles, seconds;

5) the external magnetic field does not affect the motion of the medium and the magnetic field induced by the moving medium is small compared with the external magnetic field, i.e., $\mathrm{Re}_m = \mu\sigma v l \ll 1$, where l is the characteristic length of the conducting zone, meters;

6) the active and reactive resistance of the medium between the electrodes considerably exceeds the resistance of the cylinders and the connection between them, i.e., the central and outer plates of the cylindrical capacitor are equipotential surfaces.

We note that the conditions 1-6 are satisfied simultaneously for a wide class of phenomena and considerably simplify the stated problem.

Allowance for the conditions 2 and 3 allows us to simplify the Maxwell equations describing the electromagnetic field in the investigated medium, to introduce the electrical potential $\varphi(\mathbf{E} = -\nabla\varphi)$, and to obtain div $\mathbf{j} = 0$.

From this, with allowance for the conditions 4 and 5, we obtain, in the cylindrical geometry under consideration,

$$\Delta\varphi + \frac{1}{\sigma}\frac{\partial\sigma}{\partial z_0}\frac{\partial\varphi}{\partial z_0} = 0$$

($z_0 = 0$ corresponds to the middle of the insulating spacer with a width b).

Representing the potential φ in the form $\varphi = \varphi_0 + \varphi_1$, where φ_0 is the potential in the absence of an insulating spacer and φ_1 is the distortion caused by the spacer, and satisfying the condition $b/l \ll 1$, we find that φ_1 is a solution of the Laplace equation. Physically this means that the vectors $\nabla\sigma$ and $\nabla\varphi$ are perpendicular almost everywhere inside the cylinders ($\nabla\sigma\cdot\nabla\varphi=0$): here, as in [4], the distortion of the electric field caused by the nonuniformity of conduction along the stream is unimportant. By analogy with [4], to find φ_1 we reduce the problem to a plane one by requiring that $b/R \ll 1$. Then the potential φ_1 is the solution of the Laplace equation in the upper half-plane ($x, y > 0$) with the boundary conditions

$$\varphi_1 = 0 \text{ for } |x| > b/2,\ y = 0,\ \varphi_1 \to 0 \text{ for } x^2 + y^2 \to \infty,\ \partial\varphi_1/\partial y = -(E_R + e_R) \text{ for } |x| < b/2,\ y = 0,$$

where $E_R = U(t)/(R\ln(R/r_0))$; $U(t)$ is the potential of the central electrode; r_0 is the radius of the central electrode; $e_R = nvB_R$; $B_R = (\mu/2\pi R)|I|$; $n = \text{sign}(\mathbf{v}\times\mathbf{B})$. The second boundary condition means that in this approximation the central electrode corresponds to $y=\infty$.

Using the TFKP method [9], we determine that the distortion of the electric field near the inner surface of the cylinder ($y=0$, $|x| > b/2$) is

$$E_y = (E_R + e_R)\left(1 - \frac{x}{\sqrt{x^2 - (b/2)^2}}\right).$$

The total electric field near the surface of the cylinder is $E = E_R - E_y|_{y=0}$ (a minus sign since $E_R = -E_y$). The total current to the wall of cylinder 3 is

$$i(t) = \int_0^{S(t)} j_n dS = 2\pi R \int_0^{Dt} \sigma(\xi)\,[E + e_R(\xi)]\,d\xi, \tag{2.1}$$

where D is the velocity of the wave front. In writing Eq. (2.1) it was understood that the resistance of the electrical circuit through which the current $i(t)$ flows is determined by the resistance of the conducting products.

Equation (2.1) can be rewritten in the form

$$i(t) = i_0(t) + \delta i(t) = 2\pi R \int_0^{Dt} \sigma(\xi)\,[E_R + e_R]\,d\xi + \delta i(t),$$

where

$$\delta i(t) = 2\pi R \int_0^{Dt} \sigma(\xi)\,E_y d\xi = 2\pi R \int_{b/2}^{b/2+Dt} \sigma(b/2 + Dt - x)\,E_y(x, y=0)\,dx.$$

It can be estimated that $\frac{\delta i(t)}{i_0(t)} \approx \frac{2b\sigma(z)\,[E_R + e_R(z)]}{l\,[\sigma(E_R + e_R)]_{\max}} \leqslant \frac{2b}{l}$. Here z is the coordinate in the cross section $z_0 = b/2$ separated by a distance Dt from the wave front.

Thus, $\delta i(t)/i_0(t) \ll 1$, and with a sufficient degree of accuracy

$$i(t) \simeq i_0(t) = 2\pi R \int_0^{Dt} \sigma(\xi)\,[E_R + e_R(\xi)]\,d\xi. \tag{2.2}$$

Similarly, we can show that

$$\frac{di(t)}{dt} \simeq \frac{di_0(t)}{dt} = \frac{d}{dt}2\pi R \int_0^{Dt} \sigma(\xi)\,[E_R + e_R(\xi)]\,d\xi, \tag{2.3}$$

since we have the estimate $\frac{d\delta i(t)}{dt}\Big/\frac{d}{dt}i_0(t) \leqslant \frac{2b}{l} \ll 1$.

We write Eq. (2.3) in the form

$$\frac{di_0(t)}{dt} = 2\pi R\left\{\sigma(z)D[E_R + e_R(z)] + \frac{dE_R}{dt}\int_0^{Dt}\sigma(\xi)\,d\xi + \frac{dB_R}{dt}\,n\int_0^{Dt}\sigma(\xi)v(\xi)d\xi\right\}. \tag{2.4}$$

The second and third terms on the right side of (2.4) depend on the profiles of mass velocity v and electrical conductivity σ of the medium, not known in advance, and therefore to determine v and σ we will consider two cases of electromagnetic measurements:

a) $E_R(t) = E_0 \sin\omega t$, $B_R(t) = B_0$ (E_0 and B_0 do not depend on time), and then with allowance for the condition 1 for a time t lying in the interval $t_2 - t_1 = T/2$ (where $\sin\omega t_1 = 1$ and $\sin\omega t_2 = -1$) we obtain

$$v(z) = \frac{E_0(d_1 + d_2)}{nB_0(d_1 - d_2)},\quad \sigma(z) = \frac{d_1 - d_2}{4\pi RDE_0}, \tag{2.5}$$

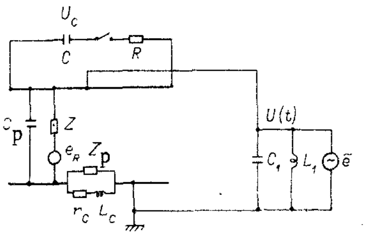

Fig. 2

where $d_j = \left.\frac{di}{dt}\right|_{t=t_j}$. By virtue of the condition 6, the entire current i(t) flows between the cylinders through the connector. The quantity di/dt can be measured, for example, by recording the voltage in a toroidal coil looping the connector or by recording the voltage developed in the connector due to its self-inductance;

b) $E_R(t) = E_0$, $B_R(t) = B_0 \sin \omega t$, and then

$$v(z) = \frac{E_0 (d_1 - d_2)}{nB_0 (d_1 + d_2)}, \quad \sigma(z) = \frac{d_1 + d_2}{4\pi RDE_0}. \tag{2.6}$$

The accuracy in determining the z coordinate in Eqs. (2.5) and (2.6) is determined by the spatial resolution, the thickness b of the spacer.

3. In the investigation of rapidly occurring processes case "a" is preferable to case "b," since the creation of powerful periodic magnetic fields is connected with far greater energy expenditures and technical difficulties than the creation of periodic electric fields.

An example of an equivalent electrical circuit for case "a" is shown in Fig. 2, where C is a capacitor for supplying the central electrode with a direct current $I \approx 10^3\text{-}10^4$ A ($RC \gg \tau$, $I = U_C/R$); e_R is the emf induced by the motion of the medium; Z and C_p are the resistance of the conducting medium and the capacitance between the central electrode and the cylinder 3 $\left(Z \sim \frac{l \ln (R/r_0)}{\pi\, \sigma_{max}},\ \frac{1}{\omega C_p} \gg Z\right)$; $\tilde{e}$ is an oscillator with a frequency ω, connected in parallel with the capacitance C_1 and the inductance L_1, for the creation of a periodic radial electric field between the electrodes $\left(ZC_1 \gg T,\ \omega \simeq \frac{1}{\sqrt{L_1 C_1}},\ R_a \ll Z\right.$, where R_a is the active resistance of the inductance L_1 and the leads); Z_p is the parasitic resistance due the closing of the space between the cylinders near the insulating spacer by the electrically conducting medium; r_c and L_c are the resistance and inductance of the connector ($Z_p \approx (2\pi R\sigma)^{-1}$, $Z_p \gg \omega L_c$, $Z \gg \omega L_c$, $Z \gg r_c$).

Thus, the capacitance C_p and the closing of the cylinders by the electrically conducting products can be neglected if one assures that $(\omega C_p)^{-1} \gg Z$ and $Z_p \approx Z$.

With variation in the polarity of the central electrode an additional alternating current $I_p \approx C_p dU(t)/dt \approx C_p \omega U_0 \approx 10^{-2}$ A develops ($\omega \lessdot 10^7$ rad/sec, $U_0 \lessdot 10\text{-}100$ V, $C_p \approx 10^{-11}$ F). Since $I_p \ll I$ ($I \approx 10^3\text{-}10^4$ A), one can assume that the magnetic field inside the cylinders is determined by the direct current I only.

In an experiment, such as in an investigation of shock waves having a steep profile behind the front, one must determine the influence of the boundary layer on the boundary conditions. For strong waves ($D \approx 1\text{-}2 \cdot 10^3$ m/sec) propagating in a gas under standard initial conditions, the flow in the boundary layer is turbulent and its thickness is $\delta_l = \beta \mathrm{Re}^{-1/5} \approx 10^{-2}R$ (for $R \approx 10^{-1}$ m and $l \approx 10^{-1}$ m), where Re and β are the Reynolds number of the stream and the coefficient for the turbulent boundary layer [10], respectively. Near the wall of the tube there is a temperature boundary layer with a low conductivity but far thinner than the dynamic boundary layer [11], so that one must expect that the influence of the contact resistance is unimportant in this case. As a result, we can take $E_{R-\delta} = E_R$ and $e_{R-\delta} = e_R$ and write the expressions for the current i(t) in the form of (2.1) and (2.2).

Then from Eqs. (2.3) and (2.5) we obtain an estimate for the error in measuring v and σ:

$$\frac{\Delta v}{v} \simeq \frac{\Delta E_0}{E_0} + \frac{\Delta B_0}{B_0} + \frac{\Delta (d_1 + d_2)}{d_1 + d_2} + \frac{\Delta (d_1 - d_2)}{d_1 - d_2}, \tag{3.1}$$

$$\frac{\Delta \sigma}{\sigma} \simeq \frac{\Delta D}{D} + \frac{\Delta E_0}{E_0} + \frac{\Delta (d_1 - d_2)}{d_1 - d_2} + \frac{2b}{l}.$$

The error in measuring each of the terms appearing in (3.1) can be made equal to 1-2%, and the proposed method allows one to determine the profiles of mass velocity and electrical conductivity from Eqs. (2.5) with an error of 5-10%.

The use of this method for detonation waves requires the analysis and estimation of the influence of the non-one-dimensionality of the wave structure leading to nonuniformity in the distribution of electrical conductivity and velocity over the tube cross section.

For a detonation in tubes with a radius $R \approx 10^{-2}$-10^{-1} m, propagating through a gas mixture and through certain heterogeneous media, the characteristic parameters usually lie in the following region: $\sigma \approx 10^{-2}$-1 Ω^{-1}/m, $l \approx 10^{-3}$-10^{-1} m, $v \approx 10^{2}$-10^{3} m/sec, $\tau \gtrsim 10^{-6}$-10^{-4} sec, $\tau_{co} \lesssim 10^{-12}$ sec. The actual sizes of cylinders are $l_{cyl} \lesssim 1$ m and the thickness of the insulating spacer is $b \approx 10^{-2}$-1 mm with the condition $b/l \approx 10^{-2}$. If one creates a magnetic field with an induction $B \approx 10^{-2}$-10^{-1} T inside the cylinders and chooses an electric field frequency of $\sim 10^{6}$-10^{7} Hz, then one can satisfy the conditions 1-6 and measure the profiles of mass velocity and electrical conductivity in a detonation wave. In this case the error must be calculated with allowance for the departure of the conditions from ideal and additional calibration on shock waves must be used.

For processes with slower variation of v and σ along the flow ($\tau > 10^{-4}$ sec) the region of measurement of the characteristic parameters of the medium is considerably expanded and the conditions for the application of the given method are improved.

LITERATURE CITED

1. J. A. Shercliff, The Theory of Electromagnetic Flow-Measurement, Univ. Press, Cambridge, Eng. (1962).
2. A. B. Vatazhin, G. A. Lyubimov, and S. A. Regirer, Magnetohydrodynamic Flows in Channels [in Russian], Nauka, Moscow (1970).
3. A. P. Ershov, P. I. Zubkov, and L. A. Luk'yanchikov, "Measurement of the width of the conductivity zone behind a detonation front in PETN," Din. Sploshnoi Sredy, No. 8 (1971).
4. A. P. Ershov, "Methods of measuring electrical conductivity behind a detonation front in condensed explosives," Din. Sploshnoi Sredy, No. 11 (1972).
5. V. V. Yakushev, "Electrical measurements in a dynamic experiment," Fiz. Goren. Vzryva, 14, No. 2 (1978).
6. A. A. Brish, M. S. Tarasov, and V. A. Tsukerman, Zh. Eksp. Teor. Fiz., 37, No. 6 (1959).
7. A. P. Ershov, "On magnetohydrodynamic methods of measuring mass velocity and electrical conductivity which vary along a stream," Zh. Prikl. Mekh. Tekh. Fiz., No. 4 (1974).
8. M. Veyssiere, Thèse de Docteur des Sciences Physique, Univ. Poitiers (1971).
9. A. G. Sveshnikov and A. N. Tikhonov, Theory of Functions of a Complex Variable [in Russian], Nauka, Moscow (1970).
10. T. V. Bazhenova and L. G. Gvozdeva, Nonsteady Interactions of Shock Waves [in Russian], Nauka, Moscow (1977).
11. R. Hill, "Preliminary investigations of the nonconducting sublayer of the boundary layer behind strong shock waves in argon," in: V. A. Popov (editor), Magnetogasdynamic Generators of Electrical Energy. Symposium 9 [in Russian], Vses. Inst. Nauch. Tekh. Inform., Moscow (1963).